Westmoreland County Public Schools
Pacing Guide and Checklist 2018-2019
Math 6 Enriched

1st Quarter

Multiplication Facts (2 days)

Vocabulary	Big Ideas/VDOE Lesson Plans	# of Days	Essential Knowledge & Skills
factors product		2	• Multiplication facts and simplifying fractions

Ratios (SOL 6.1) (3 days)

Vocabulary	Big Ideas/VDOE Lesson Plans	# of Days	Essential Knowledge & Skills
ratio relationship quantity fraction equivalent compare comparison symbolic simplest form related whole **unit rate** numerator denominator rate	**Big Ideas:** Ratios are very useful when comparing quantities in everyday life. We are asked to compare units of measure (money, speed, shapes, etc.) on daily basis. For instance, • Which is the better purchase, buying 2 candy bars for $1.79 or 3 candy bars for $2.26? • Who has the fastest average speed, John who ran 1.2 miles in 8 minutes or Julee who ran 1.5 miles in 10 minutes? • Use the ratio of girls to boys in a class in order to make girl- and boy-themed cupcakes for the grade level party. **VDOE Lesson Plan**	**3**	• Represent a relationship between two quantities using ratios. • Represent a relationship in words that make a comparison by using the notations $\frac{a}{b}$, *a:b*, and *a to b.* • Create a relationship in words for a given ratio expressed symbolically.

Understanding the Standard

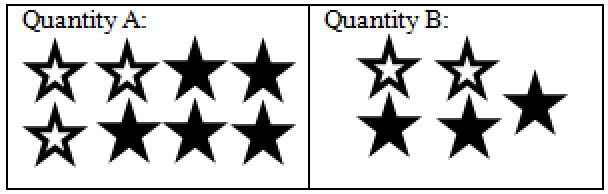
- A ratio is a comparison of any two quantities. A ratio is used to represent relationships within a quantity and between quantities. Ratios are used in practical situations when there is a need to compare quantities.
- In the elementary grades, students are taught that fractions represent a part-to-whole relationship. However, fractions may also express a measurement, an operator (multiplication), a quotient, or a ratio. Examples of fraction interpretations include:
  - Fractions as parts of wholes: $\frac{3}{4}$ represents three parts of a whole, where the whole is separated into four equal parts.
  - Fractions as measurement: the notation $\frac{3}{4}$ can be interpreted as three one-fourths of a unit.
  - Fractions as an operator: $\frac{3}{4}$ represents a multiplier of three-fourths of the original magnitude.

- Fractions as a quotient: $\frac{3}{4}$ represents the result obtained when three is divided by four.
- Fractions as a ratio: $\frac{3}{4}$ is a comparison of 3 of a quantity to the whole quantity of 4.

- A ratio may be written using a colon (a:b), the word *to* (a to b), or fraction notation $\left(\frac{a}{b}\right)$.
- The order of the values in a ratio is directly related to the order in which the quantities are compared.
  - Example: In a certain class, there is a ratio of 3 girls to 4 boys (3:4).
    Another comparison that could represent the relationship between these quantities is the ratio of 4 boys to 3 girls (4:3). Both ratios give the same information about the number of girls and boys in the class, but they are distinct ratios. When you switch the order of comparison (girls to boys vs. boys to girls), there are different ratios being expressed.
- Fractions may be used when determining equivalent ratios.
  - Example: The ratio of girls to boys in a class is 3:4, this can be interpreted as:
    $$\text{number of girls} = \frac{3}{4} \cdot \text{number of boys.}$$
    In a class with 16 boys, number of girls = $\frac{3}{4} \cdot (16) = 12$ girls.
  - Example: A similar comparison could compare the ratio of boys to girls in the class as being 4:3, which can be interpreted as:
    $$\text{number of boys} = \frac{4}{3} \cdot \text{number of girls.}$$
    In a class with 12 girls, number of boys = $\frac{4}{3} \cdot (12) = 16$ boys.
- A ratio can compare two real-world quantities (e.g., miles per gallon, unit rate, and circumference to diameter of a circle).
- Ratios may or may not be written in simplest form.
- A ratio can represent different comparisons within the same quantity or between different quantities.

Ratio	Comparison
part-to-whole (within the same quantity)	compare part of a whole to the entire whole
part-to-part (within the same quantity)	compare part of a whole to another part of the same whole
whole-to-whole (different quantities)	compare all of one whole to all of another whole
part-to-part (different quantities)	compare part of one whole to part of another whole

– Examples: Given Quantity A and Quantity B, the following comparisons could be expressed.



Ratio	Example	Ratio Notation(s)
part-to-whole (within the same quantity)	compare the number of unfilled stars to the total number of stars in Quantity A	3:8; 3 to 8; or $\frac{3}{8}$
part-to-part[1] (within the same quantity)	compare the number of unfilled stars to the number of filled stars in Quantity A	3:5 or 3 to 5
whole-to-whole[1] (different quantities)	compare the number of stars in Quantity A to the number of stars in Quantity B	8:5 or 8 to 5
part-to-part[1] (different quantities)	compare the number of unfilled stars in Quantity A to the number of unfilled stars in Quantity B	3:2 or 3 to 2

[1]Part-to-part comparisons and whole-to-whole comparisons are ratios that are not typically represented in fraction notation except in certain contexts, such as determining whether two different ratios are equivalent.

Fractions (SOL 6.2) (17 days) (No calculator!!)

Vocabulary	Big Ideas/VDOE Lesson Plans	# of Days	Essential Knowledge & Skills
fraction proper fraction improper fraction ratio numerator denominator mixed number simplify ascending descending compare order	**Big Ideas:** The more you know about the size of a number, the more efficiently you can estimate and solve when asked to figure out a total. Knowing this can be very helpful when accomplishing many practical tasks such as; using a recipe to cook for your family, computing a discount when shopping, creating a ramp for a skateboard and so on… • How can a model help show the relationship between fractions? • When is it best to use a fraction? • How can knowing where ¼, ½ and ¾ are located on a number line help me find the approximate location of	**7** 	• Represent ratios as fractions (**proper or improper**) and/or mixed numbers • Represent a fraction using an area model. (Includes shading grids) • Represent a fraction using a set model. (See curriculum framework) • Represent a fraction using a measurement model. (Number line) • Compare two fractions with denominators of 12 or less or factors of 100 using manipulatives, pictorial representations, number lines, and symbols (<, >, ≤ , ≥ , =). • (Review) Find GCF and LCM. • (Review) Improper fractions to mixed numbers and vice versa. • Order no more than four positive rational numbers expressed as

equivalent benchmarks rational numbers powers of 10 area model set model measurement model pictorial	other numbers on the number line? • How do I explain the meaning of a fraction and its numerator and denominator, and use my understanding to represent and compare fractions?		fractions (proper and improper) with denominators of 12 or less or factors of 100. Ordering may be in ascending or descending order.
decimal repeating decimal terminating decimal place value	**Big Ideas:** • How can a model help show help show the relationship between decimals? • When is it best to use a decimal? • Shopping, comparing numbers?	**2** 	• Represent a decimal using an area model. (Includes shading grids) • Represent a decimal using a set model. (See curriculum framework) • Represent a decimal using a measurement model. (Number line) • Compare two decimals through thousandths using manipulatives, pictorial representations, number lines, and symbols (<, >, ≤, ≥, =). • Order no more than four decimals (decimals through thousandths) in ascending or descending order.
percent out of 100	**Big Ideas:** • How can a model help show the relationship between percents? • When is it best to use a percent? • How can you demonstrate that one percent is greater than another? (Example: 58% is greater than 55.8%)	**2** 	• Represent a percent using an area model. (Includes shading grids) • Represent a percent using a set model. (See curriculum framework) • Represent a percent using a measurement model. (Number line) • Compare two percents using pictorial representations, and symbols (<, >, ≤, ≥, =). • Order no more than four percents in ascending or descending order.
listed above	**Big Ideas:** • How can a model help show the relationship between equivalent fractions, decimals, and percents? • When shopping, how can knowing the equivalent form of a fraction, decimal and percent help when calculating a price? Traveling? • When is it best to use a fraction? When is it best to use a decimal? When is it best to use a percent? • What is the relationship between rational numbers and their location on the number line? • How is it possible to compare/order numbers that are represented in different formats? **VDOE Lesson Plan**	**6** 	• Represent ratios as fractions **(proper or improper)**, mixed numbers, decimals, and/or percents. • Determine the decimal and percent equivalents for numbers written in fraction form **(proper or improper)** or as a mixed number, including repeating decimals. • Represent and determine equivalencies among decimals, percents, fractions **(proper or improper)**, and mixed numbers that have denominators that are 12 or less or factors of 100. • Order no more than four positive rational numbers expressed as fractions **(proper or improper)**, mixed numbers, decimals, and percents (decimals through thousandths, fractions with denominators of 12 or less or factors of 100. Ordering may be in ascending or descending order.

Understanding the Standard

- Fractions, decimals and percents can be used to represent part-to-whole ratios.
  - Example: The ratio of dogs to the total number of pets at a grooming salon is 5:8. This implies that 5 out of every 8 pets being groomed is a dog. This part-to-whole ratio could be represented as the fraction $\frac{5}{8}$ ($\frac{5}{8}$ of all pets are dogs), the decimal 0.625 (0.625 of the number of pets are dogs), or as the percent 62.5% (62.5% of the pets are dogs).
- Fractions, decimals, and percents are three different ways to express the same number. Any number that can be written as a fraction can be expressed as a terminating or repeating decimal or a percent.
- Equivalent relationships among fractions, decimals, and percents may be determined by using concrete materials and pictorial representations (e.g., fraction bars, base ten blocks, fraction circles, number lines, colored counters, cubes, decimal squares, shaded figures, shaded grids, or calculators).
- *Percent* means "per 100" or how many "out of 100"; *percent* is another name for *hundredths*.
- A number followed by a percent symbol (%) is equivalent to a fraction with that number as the numerator and with 100 as the denominator (e.g., 30% = $\frac{30}{100}$ = $\frac{3}{10}$; 139% = $\frac{139}{100}$).
- Percents can be expressed as decimals (e.g., 38% = $\frac{38}{100}$ = 0.38; 139% = $\frac{139}{100}$ = 1.39).
- Some fractions can be rewritten as equivalent fractions with denominators of powers of 10, and can be represented as decimals or percents (e.g., $\frac{3}{5}$ = $\frac{6}{10}$ = $\frac{60}{100}$ = 0.60 = 60%). Fractions, decimals, and percents can be represented by using an area model, a set model, or a measurement model. For example, the fraction $\frac{1}{3}$ is shown below using each of the three models.

- Percents are used to solve practical problems including sales, data description, and data comparison.
- The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where *a* and *b* are integers and *b* does not equal zero. The decimal form of a rational number can be expressed as a terminating or repeating decimal. A few examples of positive rational numbers are: $\sqrt{25}$, 0.275, $\frac{1}{4}$, 82, 75%, $\frac{22}{5}$, $4.\overline{59}$.
- Students are not expected to know the names of the subsets of the real numbers until grade eight.
- Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction may be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., $3\frac{5}{8}$).
- Strategies using 0, $\frac{1}{2}$ and 1 as benchmarks can be used to compare fractions.
  - Example: Which is greater, $\frac{4}{7}$ or $\frac{3}{9}$? $\frac{4}{7}$ is greater than $\frac{1}{2}$ because 4, the numerator, represents more than half of 7, the denominator. The denominator tells the number

of parts that make the whole. $\frac{3}{9}$ is less than $\frac{1}{2}$ because 3, the numerator, is less than half of 9, the denominator, which tells the number of parts that make the whole. Therefore, $\frac{4}{7} > \frac{3}{9}$.

- When comparing two fractions close to 1, use the distance from 1 as your benchmark.
  - Example: Which is greater, $\frac{6}{7}$ or $\frac{8}{9}$? $\frac{6}{7}$ is $\frac{1}{7}$ away from 1 whole. $\frac{8}{9}$ is $\frac{1}{9}$ away from 1 whole. Since, $\frac{1}{9} < \frac{1}{7}$, then $\frac{6}{7}$ is a greater distance away from 1 whole than $\frac{8}{9}$. Therefore, $\frac{6}{7} < \frac{8}{9}$.

- Some fractions such as $\frac{1}{8}$, have a decimal representation that is a terminating decimal (e. g., $\frac{1}{8} = 0.125$) and some fractions such as $\frac{2}{9}$, have a decimal representation that does not terminate but continues to repeat (e. g., $\frac{2}{9}$ = 0.222…). The repeating decimal can be written with ellipses (three dots) as in 0.222… or denoted with a bar above the digits that repeat as in $0.\overline{2}$.

Addition and Subtraction of Fractions Including Practical Problems (SOL 6.5) (7 days)

Vocabulary	Big Ideas/VDOE Lesson Plans	# of Days	Essential Knowledge & Skills
fraction simplest form mixed number improper fraction numerator denominator	**Big ideas:** We encounter rational numbers every day. • Can you think of situations involving the addition and/or the subtraction of fractions or mixed numbers?	7	• Add and subtract fractions and mixed numbers, to include like and unlike denominators, with and without regrouping, and express answers in simplest form. **(No calculator)** • Solve single-step and multistep practical problems that involve addition and subtraction with fractions **(proper or improper)** and mixed numbers that include denominators of 12 or less. Answers are expressed in simplest form. **(Calculator allowed)**

Multiplication and Division of Fractions Including Practical Problems (SOL 6.5) (8 days)

Vocabulary	Big Ideas/VDOE Lesson Plans	# of Days	Essential Knowledge & Skills
fraction mixed number improper fraction simplest form numerator denominator factor product quotient model	**Big ideas:** We encounter rational numbers every day. Do you have enough flour to double your recipe? Did your total come up correctly at the grocery store? How many servings are in a pint of Ben and Jerry's ice cream? Using flexible thinking and estimating reasonable solutions can help make quick decisions based on the information we are given. • Where in a fractions multiplication model do you find the factors? Where is the product? • How can a model be used to understand the algorithm used to multiply/divide fractions? • How can you explain that the shaded area represents the quotient when using a fraction division model? • When multiplying, is the product always bigger than the factors? Explain. • When dividing, is the quotient always smaller than the dividend? Explain. • How do I know when a result is reasonable?	8	• Demonstrate/model multiplication and division of fractions **(proper and improper)** and mixed numbers using multiple representations. **(No calculator)** • Multiply and divide fractions **(proper and improper)** and mixed numbers. Answers are expressed in simplest form. **(No calculator)** • Solve single-step and multistep practical problems that involve multiplication and division with fractions **(proper or improper)** and mixed numbers that include denominators of 12 or less. Answers are expressed in simplest form. **(Calculator allowed)**

	- How can you tell which operations are required to solve real world problems? - How do I decide what strategy will work best in a given problem situation? **VDOE Lesson Plan**		

Understanding the Standard

- A fraction can be expressed in simplest form (simplest equivalent fraction) by dividing the numerator and denominator by their greatest common factor.
- When the numerator and denominator have no common factors other than 1, then the fraction is in simplest form.
- Addition and subtraction are inverse operations as are multiplication and division.
- Models for representing multiplication and division of fractions may include arrays, paper folding, repeated addition, repeated subtraction, fraction strips, fraction rods, pattern blocks, and area models.
- It is helpful to use estimation to develop computational strategies.
  - Example: $2\frac{7}{8} \cdot \frac{3}{4}$ is about $\frac{3}{4}$ of 3, so the answer is between 2 and 3.
- When multiplying a whole number by a fraction such as $3 \cdot \frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole.
- When multiplying a fraction by a fraction such as $\frac{2}{3} \cdot \frac{3}{4}$, we are asking for part of a part.
- When multiplying a fraction by a whole number such as $\frac{1}{2} \cdot 6$, we are trying to determine a part of the whole.
- A multistep problem is a problem that requires two or more steps to solve.

Decimal Practical Problems (SOL 6.5) (4 days)

Vocabulary	Big Ideas/VDOE Lesson Plans	# of Days	Essential Knowledge & Skills
divisor dividend quotient factors product	**Big ideas:** We encounter rational numbers everyday. Do you have enough flour to double your recipe? Did your total come up correctly at the grocery store? How many servings are in a pint of Ben and Jerry's ice cream? Using flexible thinking and estimating reasonable solutions can help make quick decisions based on the information we are given. How can you use (apply) decimal operations in real life? **VDOE Lesson Plan**	**4**	Solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division with decimals expressed to thousandths with no more than two operations. **(Calculator allowed)**

Understanding the Standard

- Different strategies can be used to estimate the result of computations and judge the reasonableness of the result.
  - Example: What is an approximate answer for $2.19 \div 0.8$? The answer is around 2 because

2.19 ÷ 0.8 is about 2 ÷ 1 = 2.

- Understanding the placement of the decimal point is important when determining quotients of decimals. Examining patterns with successive decimals provides meaning, such as dividing the dividend by 6, by 0.6, and by 0.06.
- Solving multistep problems in the context of practical situations enhances interconnectedness and proficiency with estimation strategies.
- Examples of practical situations solved by using estimation strategies include shopping for groceries, buying school supplies, budgeting an allowance, and sharing the cost of a pizza or the prize money from a contest.

Quarter 1: (41 Instructional Days)

2nd Quarter

Exponents and Perfect Squares (SOL 6.4) (3 Days)

Vocabulary	Big Ideas/VDOE Lesson Plans	# of Days	Essential Knowledge & Skills
base exponents factor square root exponential notation integers perfect square power pattern place value whole numbers	**Big Ideas:** Understanding perfect squares and being able to connect them to a geometric square helps students build connections between computation and geometry. Having this understanding will make task like computing distances, finding the length of a side of a square room, and the fields of carpentry and architecture more accessible. Exponents, just like multiplication, are a shorthand way of representing another operation. Multiplication represents repeated addition, whereas exponents represent repeated multiplication. This concept plays an important part in many everyday activities. Exponents are used in measurement (square inches, cubic miles…), computer games design, computing finances, as well as population trends. • How can patterns be used to make predictions? • How does a perfect square relate to a geometric square? • How do you use powers of 10 when converting measurements in the metric system? **VDOE Lesson Plan**	**3**	• Recognize and represent patterns with bases and exponents that are whole numbers • Recognize and represent patterns of perfect squares not to exceed 20^2, by using grid paper, square tiles, tables, and calculators. • Recognize powers of 10 with whole number exponents by examining patterns in place value.

Understanding the Standard

- The symbol • can be used in grade six in place of "x" to indicate multiplication.
- In exponential notation, the base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In 8^3, 8 is the base and 3 is the exponent (e.g., $8^3 = 8 \cdot 8 \cdot 8$).
- Any real number other than zero raised to the zero power is 1. Zero to the zero power (0^0) is undefined.
- A perfect square is a whole number whose square root is an integer (e.g., $36 = 6 \cdot 6 = 6^2$). Zero (a whole number) is a perfect square.

- Perfect squares may be represented geometrically as the areas of squares the length of whose sides are whole numbers (e.g., $1 \cdot 1$, $2 \cdot 2$, $3 \cdot 3$, etc.). This can be modeled with grid paper, tiles, geoboards and virtual manipulatives.
- The examination of patterns in place value of the powers of 10 in grade six leads to the development of scientific notation in grade seven.

Integers (Model, Represent, Identify, Compare, Order, Absolute Value) (SOL 6.3) (4 days)

Vocabulary	Big Ideas/VDOE Lesson Plans	# of Days	Essential Knowledge & Skills
integers positive negative set whole number opposite order compare absolute value zero pair greater than greater than or equal to less than less than or equal to	**Big Ideas:** **6.3a,b** Integers not only show a direct relationship to some starting point, but they also give description and meaning to the numbers that occur in everyday situations. Integers are used in banking, sports, weather playing a video game, reviewing deposits or withdrawals in a checking account and even looking at weight. **6.3c** Being that distance is always a positive value, absolute value helps us calculate the distance between objects in a variety of locations. Absolute value also helps when determining elapsed time as well as comparing changes in temperature. **6.3a** • How does a number line help to compare two integers? • Are negative integers always less than positive integers? Justify your answer. **6.3b** • How does a number line help to compare two integers? • Are negative integers always less than positive integers? Justify your answer. **6.3c** • Why do we use the absolute value of a number when talking about distance? • How does the opposite of *n* differ from the absolute value of *n*? **VDOE Lesson Plan**	**4**	• Model integers, including models derived from practical situations. • Identify an integer represented by a point on a number line. • Compare and order integers using a number line. • Compare integers, using mathematical symbols (<, >, ≤ , ≥ , =). • Identify and describe the absolute value of an integer.

Understanding the Standard

- The set of integers includes the set of whole numbers and their opposites {…-2, -1, 0, 1, 2, …}. Zero has no opposite and is an integer that is neither positive nor negative.
- Integers are used in practical situations, such as temperature (above/below zero), deposits/withdrawals in a checking account, golf (above/below par), time lines, football yardage, positive and negative electrical charges, and altitude (above/below sea level).
- Integers should be explored by modeling on a number line and using manipulatives, such as two-color counters, drawings, or algebra tiles.
- The opposite of a positive number is negative and the opposite of a negative number is positive.

- Positive integers are greater than zero.
- Negative integers are less than zero.
- A negative integer is always less than a positive integer.
- When comparing two negative integers, the negative integer that is closer to zero is greater.
- An integer and its opposite are the same distance from zero on a number line.
  - Example: the opposite of 3 is −3 and the opposite of −10 is 10.
- On a conventional number line, a smaller number is always located to the left of a larger number (e.g.,–7 lies to the left of –3, thus –7 < –3; 5 lies to the left of 8 thus 5 is less than 8)

The absolute value of a number is the distance of a number from zero on the number line regardless of direction. Absolute value is represented using the symbol $|\ \ |$(e.g., $|-6| = 6$ and $|6| = 6$).

- The absolute value of zero is zero.

Integer Operations (SOL 6.6a, b) (11 days)

Vocabulary	Big Ideas/VDOE Lesson Plans	# of Days	Essential Knowledge & Skills
integer positive negative opposite set whole number sum difference product quotient division bar	**Big Ideas:** Integers not only show a direct relationship to some starting point, but they also give description and meaning to the numbers that occur in everyday situations. Integers used in banking, sports, weather. Playing a video game, reviewing deposits or withdrawals in a checking account and even looking at weight. **6.6a** • How does the knowledge of zero pairs help when modeling operations with integers? • Under what circumstances will the sum or difference of integers result in a negative solution? • Under what circumstances will the product or quotient result in a negative solution? • What strategies are most useful in helping develop algorithms for adding, subtracting, multiplying, and dividing positive and negative numbers? • Will addition of integers ever result in a sum smaller than one or smaller than both of its addends? • Will subtraction of integers ever yield a difference greater than the minuend and/or subtrahend? • When will the sum of two integers be positive? Negative? Or zero? **6.6b** • What is a real world situation in which you have to add, subtract, multiply or divide both positive and negative	**11**	• Model addition, subtraction, multiplication, and division of integers using pictorial representations or concrete manipulatives. **(No calculator)** • Add, subtract, multiply, and divide integers. **(No calculator)** • Solve practical problems involving addition, subtraction, multiplication, and division with integers. **(Calculator allowed)**

	integers? • How do you use integer operations to balance a checkbook or budget? **VDOE Lesson Plan**		

Understanding the Standard

- The set of integers is the set of whole numbers and their opposites (e.g., …-3, -2, -1, 0, 1, 2, 3…). Zero has no opposite and is neither positive nor negative.
- Integers are used in practical situations, such as temperature changes (above/below zero), balance in a checking account (deposits/withdrawals), golf, time lines, football yardage, and changes in altitude (above/below sea level).
- Concrete experiences in formulating rules for adding, subtracting, multiplying, and dividing integers should be explored by examining patterns using calculators, using a number line, and using manipulatives, such as two-color counters, drawings, or by using algebra tiles.
- Sums, differences, products and quotients of integers are either positive, negative, undefined or zero. This may be demonstrated through the use of patterns and models.

Orders of Operations Involving Integers (SOL 6.6c) (7 days) (No Calculator)

Vocabulary	Big Ideas/VDOE Lesson Plans	# of Days	Essential Knowledge & Skills
exponents power simplify grouping symbols parentheses numerical expression	**Big ideas:** • Why do we need an order of operations? • Why is multiplication and division performed from left to right? • Why is addition and subtraction performed from left to right? • If there are two different addition problems in an expression, is it ok to do the second one first? Explain using the properties of real numbers. **VDOE Lesson Plan**	**7** 	• Use the order of operations and apply properties of real numbers to simplify numerical expressions involving more than two integers. Expressions should not include braces { } or brackets [ ], but may include absolute value bars | |. Simplification will be limited to three operations, which may include simplifying a whole number raised to an exponent of 1, 2, or 3.

Understanding the Standard

- The order of operations is a convention that defines the computation order to follow in simplifying an expression. Having an established convention ensures that there is only one correct result when simplifying an expression.
- The order of operations is as follows:

  First, complete all operations within grouping symbols.[1] If there are grouping symbols within other grouping symbols, do the innermost operation first.
  Second, evaluate all exponential expressions.
  Third, multiply and/or divide in order from left to right.
  Fourth, add and/or subtract in order from left to right.

  [1]Parentheses ( ), absolute value | | (e.g., $|3(-5+2)|$), and the division bar (e.g., $\frac{3+4}{5+6}$) should be treated as grouping symbols.

- Expressions are simplified using the order of operations and applying the properties of real numbers. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of *a*, *b*, or *c* in this standard):

- Commutative property of addition: $a + b = b + a$.
- Commutative property of multiplication: $a \cdot b = b \cdot a$.
- Associative property of addition: $(a + b) + c = a + (b + c)$.
- Associative property of multiplication: $(ab)c = a(bc)$.
- Subtraction and division are neither commutative nor associative.
- Distributive property (over addition/subtraction): $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$.
- Identity property of addition (additive identity property): $a + 0 = a \; and \; 0 + a = a$.
- Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a \; and \; 1 \cdot a = a$.
- The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division.
- Inverse property of addition (additive inverse property): $a + (-a) = 0 \; and \; (-a) + a = 0$.
- Multiplicative property of zero: $a \cdot 0 = 0 \; and \; 0 \cdot a = 0$
- Substitution property: If $a = b$ then *b* can be substituted for *a* in any expression, equation or inequality.

• The power of a number represents repeated multiplication of the number (e.g., $8^3 = 8 \cdot 8 \cdot 8$). The base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In the example, 8 is the base, and 3 is the exponent.

Any number, except zero, raised to the zero power is 1. Zero to the zero power (0^0) is undefined.

Equations (SOL 6.13) (10 days) Enriched

Vocabulary	Big Ideas/VDOE Lesson Plans	# of Days	Essential Knowledge & Skills
equation term solution variable expression coefficient variable expression algebraic expression **Properties** commutative identity inverses	**Big Ideas:** Equations give us a precise way to represent many situations that arise in the world. As such, solving equations allows us to answer questions about those situations. Computers, the internet, and social media rely on solving equations to determine which search results and outcomes are best for you. Equations are used in construction to determine the amount of material required. Bankers and business workers use equations to calculate interest and determine profit. Pharmacists and doctors use equations to determine dosage of medicine. These fundamental solving skills are built upon in all future mathematics courses to address an even wider variety of practical situations. • How can a model be used to represent an equation? • How can a model be used to solve an equation? • How can you check to see if your solution is correct? • How can a practical situation be represented by a multi-step equation?	**10**	• Identify examples of the following algebraic vocabulary: equation, variable, expression, term, and coefficient. • Represent and solve one-step linear equations in one variable, using a variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale. • Apply properties of real numbers and properties of equality to solve a one-step equation in one variable. Coefficients are limited to integers and unit fractions. Numeric terms are limited to integers. • Confirm solutions to one-step linear equations in one variable. • Represent and solve a practical problem with a one-step linear equation in one variable. • Represent and solve two-step linear equations in one variable using a variety of concrete materials and pictorial representations. • Apply properties of real numbers and properties of equality to

zero addition subtraction multiplication division substitution	• How can understanding properties help when solving an equation? • How is thinking algebraically different from thinking arithmetically? • How does the knowledge of zero pairs help when solving equations? • How is an equation like a double pan balance? • How can you represent an equation in a model? Picture? • How do the properties contribute to algebraic understanding? • What strategies can be used to solve for unknowns in an algebraic expression? • How are the four basic operations related to one another? • What strategies can you use to determine if your solution is correct? • How can you use words to represent an equation or expression? • How are equations used in the real world? **VDOE Lesson Plans**		solve two-step linear equations in one variable. Coefficients and numeric terms will be rational. • Confirm algebraic solutions to linear equations in one variable. • Solve practical problems that require the solution of a two-step linear equation.

Understanding the Standard

- A one-step linear equation may include, but not be limited to, equations such as the following:
  $2x = 5$; $y - 3 = -6$; $\frac{1}{5}x = -3$; $a - (-4) = 11$.
- A variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale may be used to model solving equations in one variable.
- An expression is a representation of quantity. It may contain numbers, variables, and/or operation symbols. It does not have an "equal sign (=)" (e.g., $\frac{3}{4}$, $5x$, $140 - 38.2$, $18 \cdot 21$, $5 + x$.)
- An expression that contains a variable is a variable expression. A variable expression is like a phrase: As a phrase does not have a verb, so an expression does not have an "equal sign (=)".
  An expression cannot be solved.
- A term is a number, variable, product, or quotient in an expression of sums and/or differences. In $7x^2 + 5x - 3$, there are three terms, $7x^2$, $5x$, and 3.
- A coefficient is the numerical factor in a term. Example: in the term $3xy^2$, 3 is the coefficient; in the term z, 1 is the coefficient.
- An equation is a mathematical sentence stating that two expressions are equal.
- A variable is a symbol used to represent an unknown quantity.
- The solution to an equation is a value that makes it a true statement. Many equations have one solution and are represented as a point on a number line. Solving an equation or inequality involves a process of determining which value(s) from a specified set, if any, make the equation or inequality a true statement. Substitution can be used to determine whether a given value(s) makes an equation or inequality true.
- A two-step equation may include, but not be limited to equations such as the following:

$2x + \frac{1}{2} = -5$; $-25 = 7.2x + 1$; $\frac{x-7}{-3} = 4$; $\frac{3}{4}x - 2 = 10$.

- Properties of real numbers and properties of equality can be used to solve equations, justify equation solutions, and express simplification. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of *a*, *b*, or *c* in this standard).
- Commutative property of addition: $a + b = b + a$.
- Commutative property of multiplication: $a \cdot b = b \cdot a$.
- Subtraction and division are neither commutative nor associative.
- Identity property of addition (additive identity property): $a + 0 = a \text{ and } 0 + a = a$.
- Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a \text{ and } 1 \cdot a = a$.
- The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division.
- Inverses are numbers that combine with other numbers and result in identity elements (e.g., 5 + (–5) = 0; $\frac{1}{5} \cdot 5 = 1$).
- Inverse property of addition (additive inverse property): $a + (-a) = 0 \text{ and } (-a) + a = 0$.
- Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1 \text{ and } \frac{1}{a} \cdot a = 1$.
- Zero has no multiplicative inverse.
- Multiplicative property of zero: $a \cdot 0 = 0 \text{ and } 0 \cdot a = 0$.
- Division by zero is not a possible mathematical operation. It is undefined.
- Addition property of equality: If $a = b$, then $a + c = b + c$.
- Subtraction property of equality: If $a = b$, then $a - c = b - c$.
- Multiplication property of equality: If $a = b$, then $a \cdot c = b \cdot c$.
- Division property of equality: If $a = b \text{ and } c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
- Substitution property: If $a = b$ then *b* can be substituted for *a* in any expression, equation or inequality.

Inequalities (SOL 6.14) (5 days) Enriched

Vocabulary	Big Ideas/VDOE Lesson Plans	# of Days	Essential Knowledge & Skills
inequality variable solution set open circle closed circle greater than	**Big Ideas:** Inequalities can be used to express a range of values that can be acceptable in a given situation and graphing allows us to visualize these values. When budgeting, you are only able to spend so much without going into debt. At an amusement park, roller coasters have height restrictions based on safety regulations. Inequalities, like equations, give us a precise way to represent many situations that arise in the	5	• Given a verbal description, represent a practical situation with a one-variable linear inequality. • Apply properties of real numbers and the addition or subtraction property of inequality to solve a one-step linear inequality in one variable, and graph the solution on a number line. Numeric terms being added or subtracted from the variable are limited to integers.

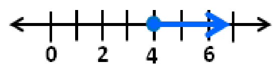
greater than or equal to less than less than or equal to **<u>Properties</u>** commutative identity inverses zero addition subtraction substitution	world and solving inequalities allows us to answer questions about those situations. **6.14a** • How can you compare and contrast the process for solving an equation and inequality. • How can you represent an inequality in a model? Picture? **6.14b** • How can you compare and contrast the solutions to an equation and inequalities. • When are inequalities used in the real world? • How can we tell if a solution should be graphed with an open or closed circle? • How can we tell if a number is a solution from looking at the graph? Equation? • How do the properties contribute to solving inequalities? **VDOE Lesson Plans**		• Given the graph of a linear inequality with integers, represent the inequality two different ways (e.g. $x < -5$ or $-5 > x$) using symbols. • Identify a numeric value(s) that is part of the solution set of a given inequality. • Apply properties of real numbers and the multiplication and division properties of inequality to solve one-step inequalities in one variable, and the addition, subtraction, multiplication, and division properties of inequality to solve two-step inequalities in one variable. Coefficients and numeric terms will be rational. • Represent solutions to inequalities algebraically and graphically using a number line. • Solve practical problems that require the solution of a one- or two-step inequality. • Identify a numerical value(s) that is part of the solution set of a given inequality.

Understanding the Standard

- The solution set to an inequality is the set of all numbers that make the inequality true.
- Inequalities can represent practical situations.

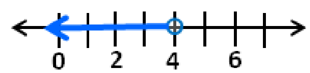
  Example: Jaxon works at least 4 hours per week mowing lawns. Write an inequality representing this situation and graph the solution.

  $x \geq 4$ or $4 \leq x$

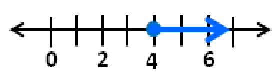


  Students might then be asked: “Would Jaxon ever work 3 hours in a week? 6 hours?

- The variable in an inequality may represent values that are limited by the context of the problem or situation. Example: if the variable represents all children in a classroom who are taller than 36 inches, the variable will be limited to have a minimum and maximum value based on the heights of the children. Students are not expected to represent these situations with a compound inequality (e.g., $36 < x < 70$) but only recognize that the values satisfying the single inequality ($x > 36$) will be limited by the context of the situation.
- Inequalities using the $<$ or $>$ symbols are represented on a number line with an open circle on the number and a shaded line over the solution set.
  Example: When graphing $x < 4$, use an open circle above the 4 to indicate that the 4 is not included.



- Inequalities using the $\leq$ or $\geq$ symbols are represented on a number line with a closed circle on the number and shaded line in the direction of the solution set.
  Example: When graphing $x \geq 4$ fill in the circle above the 4 to indicate that the 4 is included.



- It is important for students to see inequalities written with the variable before the inequality symbol and after. Example: $x > 5$ is not the same relationship as $5 > x$. However, $x > 5$ is the same relationship as $5 < x$.

- A one-step linear inequality may include, but not be limited to, inequalities such as the following: $2 + x > 5$; $y - 3 \leq -6$; $a - (-4) \geq 11$.

- Solving an equation or inequality involves a process of determining which value(s) from a specified set, if any, make the equation or inequality a true statement. Substitution can be used to determine whether a given value(s) makes an equation or inequality true.

- Properties of real numbers and properties of inequality can be used to solve inequalities, justify solutions, and express simplification. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of *a*, *b*, or *c* in this standard):

  - Commutative property of addition: $a + b = b + a$.
  - Commutative property of multiplication: $a \cdot b = b \cdot a$.
  - Subtraction and division are neither commutative nor associative.
  - Identity property of addition (additive identity property): $a + 0 = a \text{ and } 0 + a = a$.
  - Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a \text{ and } 1 \cdot a = a$.
  - The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division.
  - Inverses are numbers that combine with other numbers and result in identity elements (e.g., $5 + (-5) = 0$; $\frac{1}{5} \cdot 5 = 1$).
  - Inverse property of addition (additive inverse property): $a + (-a) = 0 \text{ and } (-a) + a = 0$.
  - Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1 \text{ and } \frac{1}{a} \cdot a = 1$.
  - Zero has no multiplicative inverse.
  - Multiplicative property of zero: $a \cdot 0 = 0 \text{ and } 0 \cdot a = 0$.

  - Addition property of inequality: If $a < b$, then $a + c < b + c$; if $a > b$, then $a + c > b + c$ (this property also applies to $\leq \text{ and } \geq$).
  - Subtraction property of inequality: If $a < b$, then $a - c < b - c$; if $a > b$, then $a - c > b - c$ (this property also applies to $\leq \text{ and } \geq$).
  - Multiplication property of inequality: If $a < b \text{ and } c > 0$, then $a \cdot c < b \cdot c$; if $a > b \text{ and } c > 0$, then $a \cdot c > b \cdot c$.
  - Multiplication property of inequality (multiplication by a negative number): If $a < b \text{ and } c < 0$, then $a \cdot c > b \cdot c$; if $a > b \text{ and } c < 0$, then $a \cdot c < b \cdot c$.
  - Division property of inequality: If $a < b \text{ and } c > 0$, then $\frac{a}{c} < \frac{b}{c}$; if $a > b \text{ and } c > 0$, then $\frac{a}{c} > \frac{b}{c}$.
  - Division property of inequality (division by a negative number): If $a < b \text{ and } c < 0$, then

$\frac{a}{c} > \frac{b}{c}$; if $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$.

– Substitution property: If $a = b$ then b can be substituted for a in any expression, equation or inequality.

Quarter 2: (40 Instructional Days)

3rd Quarter

Verbal Statements (SOL 6.13) (3 Days) Enriched

Vocabulary	Big Ideas/VDOE Lesson Plans	# of Days	Essential Knowledge & Skills
sum difference product quotient more than less than twice increased by decreased	**Essential Questions:** • How can you use words to represent an equation or expression? **VDOE Lesson Plans**	**3**	• Write verbal expressions and sentences as algebraic expressions and equations. • Write algebraic expressions and equations as verbal expressions and sentences. • Write verbal expressions and sentences as algebraic expressions and inequalities. • Write algebraic expressions and inequalities as verbal expressions and sentences.

Understanding the Standard

- An expression that contains a variable is a variable expression. A variable expression is like a phrase: As a phrase does not have a verb, so an expression does not have an "equal sign (=)".
  An expression cannot be solved.
- A verbal expression can be represented by a variable expression. Numbers are used when they are known; variables are used when the numbers are unknown. Example, the verbal expression "a number multiplied by 5" could be represented by the variable expression "$n \cdot 5$" or "$5n$."
- An algebraic expression is a variable expression that contains at least one variable (e.g., $x - 3$).
- A verbal sentence is a complete word statement (e.g., "The sum of a number and two is five" could be represented by "$n + 2 = 5$").
- An algebraic equation is a mathematical statement that says that two expressions are equal
  (e.g., $2x = 7$).

Coordinate Plane (SOL 6.8) (5 Days)

Vocabulary	Big Ideas/VDOE Lesson Plans	# of Days	Essential Knowledge & Skills
coordinate plane first coordinate x-axis horizontal second coordinate y-axis vertical origin	**Big ideas:** Graphing on a coordinate plane is useful for reading maps. Like the coordinates of an ordered pair, giving directions requires attention to direction of movement and magnitude of that movement. Air traffic control, satellites, and the military all use concepts of a coordinate plane to identify precise locations. **6.8a** • How are the axes of a coordinate plane related to a number line?	**5**	• Identify and label the axes, origin, and quadrants of a coordinate plane. • Identify the quadrant or the axis on which a point is positioned by examining the coordinates (ordered pairs) of the point. Ordered pairs will be limited to coordinates expressed as integers. • Graph ordered pairs in the four quadrants and on the axes of a coordinate plane. Ordered pairs will be limited to coordinates expressed as integers. • Identify ordered pairs represented by points in the four quadrants

quadrants coordinates ordered pair Roman numerals I, II, III, IV polygons vertices distance	**6.8b** • How is the concept of the coordinate plane applied in practical situations? (i.e. maps) • On a coordinate plane, how are points on a horizontal line related to each other? • On a coordinate plane, how are points on a vertical line related to each other? • How do you determine the distance a point is from an axis? **VDOE Lesson Plans**		and on the axes of the coordinate plane. Ordered pairs will be limited to coordinates expressed as integers. • Relate the coordinates of a point to the distance from each axis and relate the coordinates of a single point to another point on the same horizontal or vertical line. Ordered pairs will be limited to coordinates expressed as integers. • Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to determine the length of a side joining points with the same first coordinate or the same second coordinate. Ordered pairs will be limited to coordinates expressed as integers. Apply these techniques in the context of solving practical and mathematical problems.

Understanding the Standard

- In a coordinate plane, the coordinates of a point are typically represented by the ordered pair (*x*, *y*), where *x* is the first coordinate and *y* is the second coordinate.
- Any given point is defined by only one ordered pair in the coordinate plane.
- The grid lines on a coordinate plane are perpendicular.
- The axes of the coordinate plane are the two intersecting perpendicular lines that divide it into its four quadrants. The *x*-axis is the horizontal axis and the *y*-axis is the vertical axis.
- The quadrants of a coordinate plane are the four regions created by the two intersecting perpendicular lines (*x*- and *y*-axes). Quadrants are named in counterclockwise order. The signs on the ordered pairs for quadrant I are (+,+); for quadrant II, (–,+); for quadrant III, (–, –); and for quadrant IV, (+,–).
- In a coordinate plane, the origin is the point at the intersection of the *x*-axis and *y*-axis; the coordinates of this point are (0, 0).
- For all points on the *x*-axis, the *y*-coordinate is 0. For all points on the *y*-axis, the *x*-coordinate is 0.
- The coordinates may be used to name the point. (e.g., the point (2, 7)). It is not necessary to say "the point whose coordinates are (2, 7)." The first coordinate tells the location or distance of the point to the left or right of the *y*-axis and the second coordinate tells the location or distance of the point above or below the *x*-axis. For example, (2, 7) is two units to the right of the *y*-axis and seven units above the *x*-axis.
- Coordinates of points having the same *x*-coordinate are located on the same vertical line. For example, (2, 4) and (2, -3) are both two units to the right of the *y*-axis and are vertically seven units from each other.
- Coordinates of points having the same *y*-coordinate are located on the same horizontal line. For example, (-4, -2) and (2, -2) are both two units below the *x*-axis and are horizontally six units from each other.

Proportional Relationships (SOL 6.12) (8 Days)

Vocabulary	Big Ideas/VDOE Lesson Plans	# of Days	Essential Knowledge & Skills
ratio unit rate ratio table multiple relationships	**Big ideas:** Proportional reasoning involves thinking about relationships and making comparisons of quantities or values. People use proportional reasoning to calculate best buys, taxes and investments, to work with drawings and maps, to measure or exchange money, to adjust recipes,		**In this unit, make connections to the ratio unit previously taught.** • Make a table of equivalent ratios to represent a proportional relationship between two quantities, when given a ratio. • Make a table of equivalent ratios to represent a proportional relationship between two quantities, when given a practical

equivalent equivalent ratio proportion proportional relationship input/output table graph ordered pairs coordinate plane multiple representations constant double number lines	or to create various concentrations of mixtures and solutions. **6.12a** • What is the relationship between a ratio and a proportion? • How does a proportion compare to two equivalent ratios? • How would you use a table to show a proportional relationship? • What is a situation that demonstrates a proportional relationship? • What is a situation that does not demonstrate a proportional relationship? **6.12b** • When would a unit rate be useful to solve a problem? • How do you interpret a unit rate? • What are the similarities between simplifying a fraction and showing proportional relationship? **6.12c** • What is a proportional relationship? **6.12d** • How is a proportional relationship, when written in words, shown in a ratio table and a graph? **VDOE Lesson Plans**	**8**	situation. • Identify the unit rate of a proportional relationship represented by a table of values or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. • Determine a missing value in a ratio table that represents a proportional relationship between two quantities using a unit rate. Unit rates are limited to positive values. • Determine whether a proportional relationship exists between two quantities, when given a table of values or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. • Determine whether a proportional relationship exists between two quantities given a graph of ordered pairs. Unit rates are limited to positive values. • Make connections between and among multiple representations of the same proportional relationship using verbal descriptions, ratio tables, and graphs. Unit rates are limited to positive values.

Understanding the Standard

- A ratio is a comparison of any two quantities. A ratio is used to represent relationships within a quantity and between quantities.
- Equivalent ratios arise by multiplying each value in a ratio by the same constant value. For example, the ratio of 4:2 would be equivalent to the ratio 8:4, since each value in the first ratio could be multiplied by 2 to obtain the second ratio.
- A proportional relationship consists of two quantities where there exists a constant number (constant of proportionality) such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity.
- Proportional thinking requires students to thinking multiplicatively, versus additively. The relationship between two quantities could be additive (i.e., one quantity is a result of adding a value to the other quantity) or multiplicative (i.e.., one quantity is the result of multiplying the other quantity by a value). Therefore, it is important to use practical situations to model proportional relationships, because context can help students to see the relationship. Students will explore algebraic representations of additive relationships in grade seven.

– Example:

Additive relationship:	
x	y
2 $\xrightarrow{+8}$	10
3 $\xrightarrow{+8}$	11
4 $\xrightarrow{+8}$	12
5 $\xrightarrow{+8}$	13

Multiplicative relationship:	
x	y
2 $\xrightarrow{\cdot 5}$	10
3 $\xrightarrow{\cdot 5}$	15
4 $\xrightarrow{\cdot 5}$	20
5 $\xrightarrow{\cdot 5}$	25

- In the additive relationship, *y* is the result of adding 8 to *x*.
- In the multiplicative relationship, *y* is the result of multiplying 5 times *x*.
- The ordered pair (2, 10) is a quantity in both relationships; however, the relationship evident between the other quantities in the table, discerns between additive or multiplicative.

- Students have had experiences with tables of values (input/output tables that are additive and multiplicative) in elementary grades.
- A ratio table is a table of values representing a proportional relationship that includes pairs of values that represent equivalent rates or ratios. A constant exists that can be multiplied by the measure of one quantity to get the measure of the other quantity for every ratio pair. The same proportional relationship exists between each pair of quantities in a ratio table.
  – Example: Given that the ratio of *y* to *x* in a proportional relationship is 8:4, create a ratio table that includes three additional equivalent ratios.

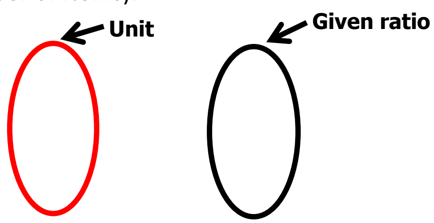
x		y
1	$\cdot 2$	2
2	$\cdot 2$	4
3	$\cdot 2$	6
4	$\cdot 2$	8
5	$\cdot 2$	10

← Ratio that is given

Students have had experience with tables of values (input/output tables) in elementary grades and the concept of a ratio table should be connected to their prior knowledge of representing number patterns in tables.

- A rate is a ratio that involves two different units and how they relate to each other. Relationships between two units of measure are also rates (e.g., inches per foot).

- A unit rate describes how many units of the first quantity of a ratio correspond to one unit of the second quantity.
  - Example: If it costs \$10 for 5 items at a store (a ratio of 10:5 comparing cost to the number of items), then the unit rate would be \$2.00/per item (a ratio of 2:1 comparing cost to number of items).



# of items (*x*)	1	2	5	10
Cost in \$ (*y*)	\$2.00	\$4.00	\$10.00	\$20.00

- Any ratio can be converted into a unit rate by writing the ratio as a fraction and then dividing the numerator and denominator each by the value of the denominator. Example: It costs \$8 for 16 gourmet cookies at a bake sale. What is the price per cookie (unit rate) represented by this situation?

$$\frac{8}{16} = \frac{8 \div 16}{16 \div 16} = \frac{0.5}{1}$$

So, it would cost \$0.50 per cookie, which would be the unit rate.

  - Example: $\frac{8}{16}$ and 40 to 10 are ratios, but are not unit rates. However, $\frac{0.5}{1}$ and 4 to 1 are unit rates.

- Students in grade six should build a conceptual understanding of proportional relationships and unit rates before moving to more abstract representations and complex computations in higher grade levels. Students are not expected to use formal calculations for slope and unit rates (e.g., slope formula) in grade six.

  - Example of a proportional relationship:

Ms. Cochran is planning a year-end pizza party for her students. Ace Pizza offers free delivery and charges \$8 for each medium pizza. This ratio table represents the cost (*y*) per number of pizzas ordered (*x*).

x number of pizzas	1	2	3	4
y total cost	8	16	24	32

In this relationship, the ratio of *y* (cost in \$) to *x* (number of pizzas) in each ordered pair is the same:

$$\frac{8}{1} = \frac{16}{2} = \frac{24}{3} = \frac{32}{4}$$

  - Example of a non-proportional relationship:

Uptown Pizza sells medium pizzas for \$7 each but charges a \$3 delivery fee per order. This table represents the cost per number of pizzas ordered.

x number of pizzas	1	2	3	4
y total cost	10	17	24	31

The ratios represented in the table above are not equivalent.

In this relationship, the ratio of y to x in each ordered pair is not the same:

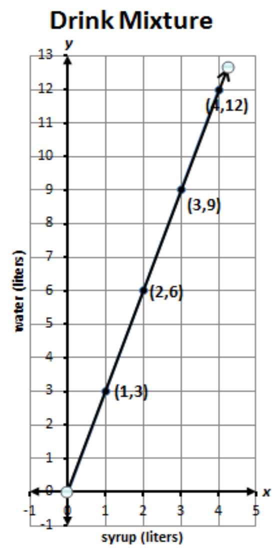
$$\frac{10}{1} \neq \frac{17}{2} \neq \frac{24}{3} \neq \frac{31}{4}$$

Other non-proportional relationships will be studied in later mathematics courses.

- Proportional relationships can be described verbally using the phrases "for each," "for every," and "per."
- Proportional relationships involve collections of pairs of equivalent ratios that may be graphed in the coordinate plane. The graph of a proportional relationship includes ordered pairs (x, y) that represent pairs of values that may be represented in a ratio table.
- Proportional relationships can be expressed using verbal descriptions, tables, and graphs.
  - Example: (verbal description) To make a drink, mix 1 liter of syrup with 3 liters of water. If x represents how many liters of syrup are in the mixture and y represents how many liters of water are in the mixture, this proportional relationship can be represented using a ratio table:

Syrup (liters) x	1	2	3	4
Water (liters) y	3	6	9	12

The ratio of the amount of water (y) to the amount of syrup (x) is 3:1. Additionally, the proportional relationship may be graphed using the ordered pairs in the table.

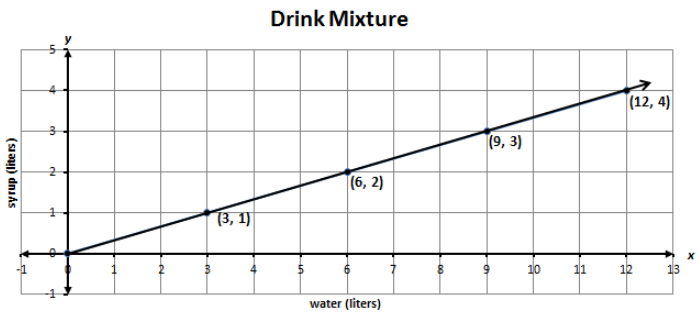


- The representation of the ratio between two quantities may depend upon the order in which the quantities are being compared.
    - Example: In the mixture example above, we could also compare the ratio of the liters of syrup per liters of water, as shown:

Water (liters) x	3	6	9	12
Syrup (liters) y	1	2	3	4

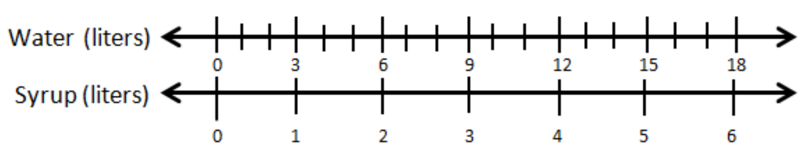
In this comparison, the ratio of the amount of syrup (y) to the amount of water (x) would be 1:3.

The graph of this relationship could be represented by:



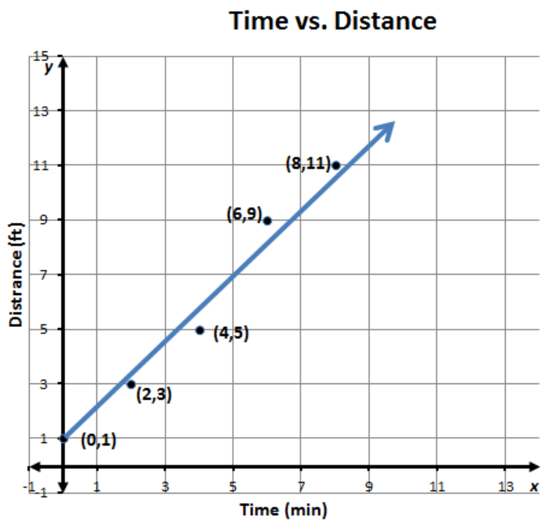
Students should be aware of how the order in which quantities are compared affects the way in which the relationship is represented as a table of equivalent ratios or as a graph.

- Double number line diagrams can also be used to represent proportional relationships and create collections of pairs of equivalent ratios.
    - Example:



In this proportional relationship, there are three liters of water for each liter of syrup represented on the number lines.

- A graph representing a proportional relationship includes ordered pairs that lie in a straight line that, if extended, would pass through (0, 0), creating a pattern of horizontal and vertical increases. The context of the problem and the type of data being represented by the graph must be considered when determining whether the points are to be connected by a straight line on the graph.
    - Example of the graph of a non-proportional relationship:



The relationship of distance (y) to time (x) is non-proportional. The ratio of y to x for each ordered pair is not equivalent. That is,

$$\frac{11}{8} \neq \frac{9}{6} \neq \frac{5}{4} \neq \frac{3}{2} \neq \frac{1}{0}$$

The points of the graph do not lie in a straight line. Additionally, the line does not pass through the point (0, 0), thus the relationship of y to x cannot be considered proportional.

- Practical situations that model proportional relationships can typically be represented by graphs in the first quadrant, since in most cases the values for x and y are not negative.
- Unit rates are not typically negative in practical situations involving proportional relationships.
- A unit rate could be used to find missing values in a ratio table.
  - Example: A store advertises a price of $25 for 5 DVDs. What would be the cost to purchase 2 DVDs? 3 DVDs? 4 DVDs?

# DVDs	1	2	3	4	5
Cost	$5	?	?	?	$25

The ratio of $25 per 5 DVDs is also equivalent to a ratio of $5 per 1 DVD, which would be the unit rate for this relationship. This unit rate could be used to find the missing value of the ratio table above. If we multiply the number of DVDs by a constant multiplier of 5 (the unit rate) we obtain the total cost. Thus, 2 DVDs would cost $10, 3 DVDs would cost $15, and 4 DVDs would cost $20.

Functions (7.10) (4 Days) Enriched

Vocabulary	Big Ideas/VDOE Lesson Plans	# of Days	Essential Knowledge & Skills
slope rate of change	**Big Ideas** • Proportional reasoning involves thinking about relationships and making comparisons of quantities or values. People use	**4**	• Determine the slope, *m*, as rate of change in a proportional relationship between two quantities given a table of values or a

proportional relationship multiplicative y = mx ordered pair y-intercept additive relationship additive y = x + b line equation tables graphs vertical horizontal rate origin dependent variable independent variable constant	proportional reasoning to calculate best buys, taxes and investments, to work with drawings and maps, to measure or exchange money, to adjust recipes, or to create various concentrations of mixtures and solutions. • The slope of a line represents a constant rate of change. Many practical situations including science, construction, and business all represent various situations in terms of rate of change. In addition, rate of change is the foundation of calculus where interpretation is also essential with correct units of measure. **7.10a** • What relationship exists between the slope of a line and a table of ordered pairs? • Where is slope present in real life? • How can any two ordered pairs of a function be used to find the slope? • How can you represent a relationship using the equation y = mx? **7.10b** • How can a line represent a proportional relationship? • What connection can be made between the equation y = mx and the line it represents? **7.10c** • What does interception mean? • When given a practical problem, what does they-intercept represent? • What is the association between the equation y = x + b and the y intercept for the same relationship? **7.10d** • How would you describe an additive relationship? • Given a graph of a line, how could you determine if it represented an additive relationship? **7.10e** • What relationships can exists between and among verbal descriptions, tables, equations, and graphs? **VDOE Lesson Plans**		verbal description, including those represented in a practical situation, and write an equation in the form $y = mx$ to represent the relationship. Slope will be limited to positive values. • Graph a line representing a proportional relationship, between two quantities given an ordered pair on the line and the slope, m, as rate of change. Slope will be limited to positive values. • Graph a line representing a proportional relationship between two quantities given the equation of the line in the form $y = mx$, where m represents the slope as rate of change. Slope will be limited to positive values. • Determine the y-intercept, b, in an additive relationship between two quantities given a table of values or a verbal description, including those represented in a practical situation, and write an equation in the form $y = x + b$, $b \neq 0$, to represent the relationship. • Graph a line representing an additive relationship ($y = x + b$, $b \neq 0$) between two quantities, given an ordered pair on the line and the y-intercept (b). The y-intercept (b) is limited to integer values and slope is limited to 1. • Graph a line representing an additive relationship between two quantities, given the equation in the form $y = x + b$, $b \neq 0$. The y-intercept (b) is limited to integer values and slope is limited to 1. • Make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

Understanding the Standard

- When two quantities, x and y, vary in such a way that one of them is a constant multiple of the other, the two quantities are “proportional”. A model for that situation is $y = mx$ where m is the slope or rate of change. Slope may also represent the unit rate of a proportional relationship between two quantities, also referred to as the constant of proportionality or the constant ratio of y to x.

- The slope of a proportional relationship can be determined by finding the unit rate.

  Example: The ordered pairs (4, 2) and (6, 3) make up points that could be included on the graph of a proportional relationship. Determine the slope, or rate of change, of a line passing through these points. Write an equation of the line representing this proportional relationship.

x	***y***
4	2
6	3

The slope, or rate of change, would be $\frac{1}{2}$ or 0.5 since the *y*-coordinate of each ordered pair would result by multiplying $\frac{1}{2}$ times the *x*-coordinate. This would also be the unit rate of this proportional relationship. The ratio of *y* to *x* is the same for each ordered pair. That is, $\frac{y}{x} = \frac{2}{4} = \frac{3}{6} = \frac{1}{2} = 0.5$

The equation of a line representing this proportional relationship of *y* to *x* is $y = \frac{1}{2}x$ or $y = 0.5x$.

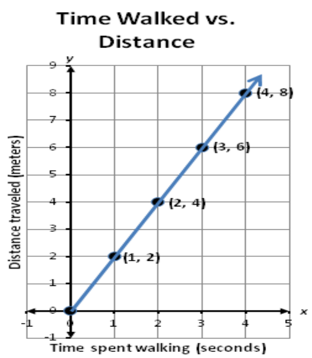
  - The slope of a line is a rate of change, a ratio describing the vertical change to the horizontal change of the line.

  slope = $\frac{change\ in\ y}{change\ in\ x} = \frac{vertical\ change}{horizontal\ change}$

- The graph of the line representing a proportional relationship will include the origin (0, 0).
- A proportional relationship between two quantities can be modeled given a practical situation. Representations may include verbal descriptions, tables, equations, or graphs. Students may benefit from an informal discussion about independent and dependent variables when modeling practical situations. Grade eight mathematics formally addresses identifying dependent and independent variables.
  - Example (using a table of values): Cecil walks 2 meters every second (verbal description). If *x* represents the number of seconds and *y* represents the number of meters he walks, this proportional relationship can be represented using a table of values:

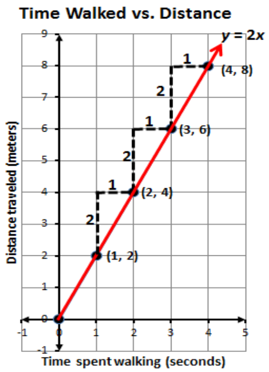
x (seconds)	1	2	3	4
y (meters)	2	4	6	8

  This proportional relationship could be represented using the equation $y = 2x$, since he walks 2 meters for each second of time. That is, $\frac{y}{x} = \frac{2}{1} = \frac{4}{2} = \frac{6}{3} = \frac{8}{4} = 2$, the unit rate (constant of proportionality) is 2 or $\frac{2}{1}$. The same constant ratio of *y* to *x* exists for every ordered pair. This proportional relationship could be represented by the following graph:



  - A graph of a can proportional relationship be created by graphing ordered pairs generated in a table of values (as shown above), or by observing the rate of change or slope of the relationship and using slope triangles to graph ordered pairs that satisfy the relationship given.

– Example (using slope triangles): Cecil walks 2 meters every second. If *x* represents the number of seconds and *y* represents the number of meters he walks, this proportional relationship can be represented graphically using slope triangles.



The rate of change from (1, 2) to (2, 4) is 2 units up (the change in *y*) and 1 unit to the right (the change in *x*), $\frac{2}{1}$ or 2. Thus, the slope of this line is 2. Slope triangles can be used to generate points on a graph that satisfy this relationship.

- Proportional thinking requires students to thinking multiplicatively. However, the relationship between two quantities is not always proportional. The relationship between two quantities could be additive (i.e., one quantity is a result of adding a value to the other quantity) or multiplicative (i.e.., one quantity is the result of multiplying the other quantity by a value). Therefore, it is important to use practical situations to model proportional relationships, since context can help students to see the relationship.

Additive relationship:

x	y
2 $\xrightarrow{+8}$	10
3 $\xrightarrow{+8}$	11
4 $\xrightarrow{+8}$	12
5 $\xrightarrow{+8}$	13

Multiplicative relationship:

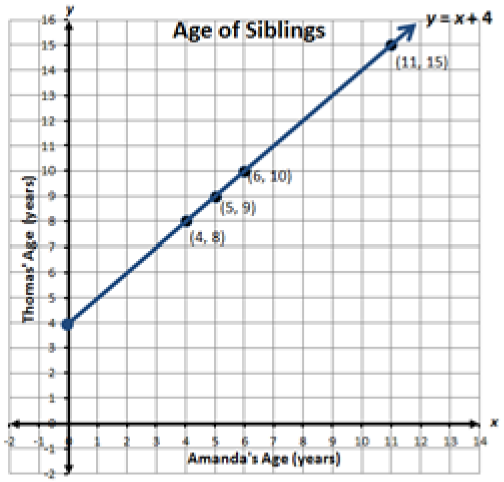
x	y
2 $\xrightarrow{\cdot 5}$	10
3 $\xrightarrow{\cdot 5}$	15
4 $\xrightarrow{\cdot 5}$	20
5 $\xrightarrow{\cdot 5}$	25

In the additive relationship, *y* is the result of adding 8 to *x*.
In the multiplicative relationship, *y* is the result of multiplying 5 times *x*.
The ordered pair (2, 10) is a quantity in both relationships, however, the relationship evident between the other quantities in the table, discerns between additive or multiplicative.

- Two quantities, *x* and *y*, have an additive relationship when a constant value, *b*, exists where $y = x + b$, where $b \neq 0$. An additive relationship is not proportional and its graph does not pass through (0, 0). Note that *b* can be a positive value or a negative value. When *b* is negative, the right side of the equation could be written using a subtraction symbol (e.g., if *b* is –5, then the equation $y = x - 5$ could be used).
  – Example: Thomas is four years older than his sister, Amanda (verbal description). The following table shows the relationship between their ages at given points in time.

Amanda's Age	4	5	6	11
Thomas' Age	8	9	10	15

The equation that represents the relationship between Thomas' age and Amanda's age is $y = x + 4$. A graph of the relationship between their ages is shown below:



- Graphing a line given an equation can be addressed using different methods. One method involves determining a table of ordered pairs by substituting into the equation values for one variable and solving for the other variable, plotting the ordered pairs in the coordinate plane, and connecting the points to form a straight line. Another method involves using slope triangles to determine points on the line.
  - Example: Graph the equation $y = x - 1$.
    In order to graph the equation, we can create a table of values by substituting arbitrary values for x to determine coordinating values for y:

x	x - 1	y
-1	(-1) - 1	-2
0	(0) - 1	-1
1	(1) - 1	0
2	(2) - 1	1

These values can then be plotted as the points
(-1, -2), (0, -1), (1, 0), and (2, 1) on a graph.

An equation written in $y = x + b$ form provides information about the graph. If the equation is $y = x - 1$, then the slope, m, of the line is 1 or $\frac{1}{1}$ and the point where the line crosses the y-axis can be located at (0, -1). We also know,

slope = $m = \frac{change\ in\ y-value}{change\ in\ x-value} = \frac{+1}{+1}\ or\ \frac{-1}{-1}$

So we can plot some other points on the graph using this relationship between y and x values.

A table of values can be used to determine the graph of a line. The *y*-intercept is located on the *y*-axis which is where the *x*-coordinate is 0. The change in each *y*-value compared to the corresponding *x*-value can be verified by the patterns in the table of values.

x	*y*
-1	-2
0	-1
1	0
2	1

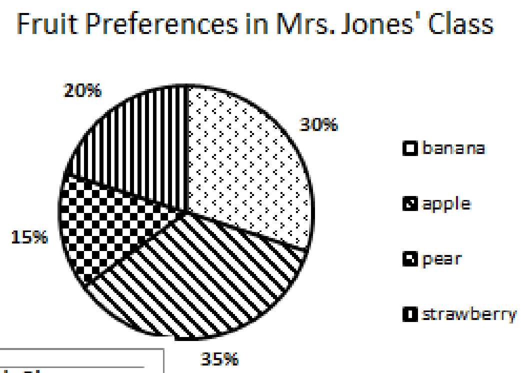
Circle Graphs (SOL 6.10) (8 Days)

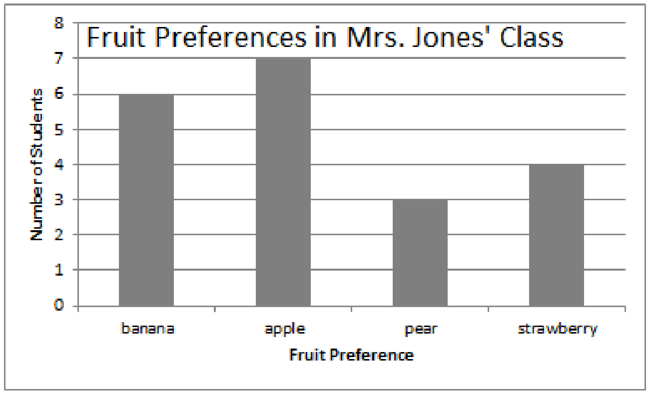
Vocabulary	Big Ideas/VDOE Lesson Plans	# of Days	Essential Knowledge & Skills
circle graph data percentage observation inference bar graph pictograph lie plot predictions	**Big ideas:** • Circle graphs are used in everyday life to help people easily organize information as parts of a whole to help make decisions or comparisons. • In many practical situations in a variety of areas such as business and science, observational data is gathered and graphed so that a unifying model can be determined to make predictions about unobserved input values. These predictions assist researchers in making important decisions about the situation of study. Therefore, it is also important that the reasonableness of predictions be addressed. **6.10a** • How does a circle graph help me organize my data? **6.10b** • How do people use data to influence others? • Is it possible to manipulate data to change the way the data is perceived? **6.10c** • How does the type of data influence the choice of graph? **VDOE Lesson Plans**	**8**	• Collect, organize, and represent data in a circle graph. The number of data values should be limited to allow for comparisons that have denominators of 12 or less or those that are factors of 100 (e.g., in a class of 20 students, 7 choose apples as a favorite fruit, so the comparison is 7 out of 20, $\frac{7}{20}$, 35%). • Make observations and inferences about data represented in a circle graph. • Compare data represented in a circle graph with the same data represented in bar graphs, pictographs, and line plots.

Understanding the Standard

- Circle graphs are used for data showing a relationship of the parts to the whole.
  - Example: the favorite fruit of 20 students in Mrs. Jones class was recorded in the table. Compare the same data displayed in both a circle graph and a bar graph.

Fruit Preference	# of students
banana	6
apple	7
pear	3
strawberry	4





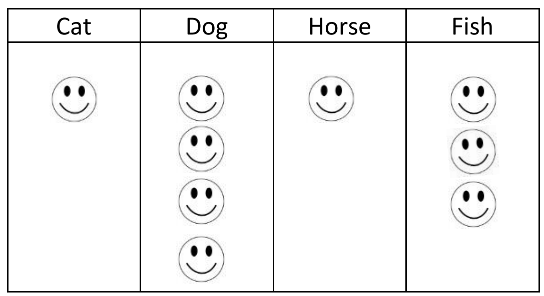
- Circle graphs can represent percent or frequency.
- Circle graphs are not useful for representing data with large numbers of categories.
- Teachers should be reasonable about the selection of data values. The number of data values can affect how a circle graph is constructed (e.g., 10 out of 25 would be 40%, but 7 out of 9 would be $77.\overline{7}\%$, making the construction of a circle graph more complex). Students should have experience constructing circle graphs, but a focus should be placed on the analysis of circle graphs.
- Students are not expected to construct circle graphs by multiplying the percentage of data in a category by 360° in order to determine the central angle measure. Limiting comparisons to fraction parameters noted will assist students in constructing circle graphs.
- To collect data for any problem situation, an experiment can be designed, a survey can be conducted, or other data-gathering strategies can be used. The data can be organized, displayed, analyzed, and interpreted to solve the problem.
- Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data.
- Different types of graphs can be used to display categorical data. The way data are displayed often depends on what someone is trying to communicate.

A line plot is used for categorical and discrete numerical data and is used to show frequency of data on a number line. It is a simple way to organize data.

Example:

- A bar graph is used for categorical and discrete numerical data (e.g., number of months or number of people with a particular eye color) and is used to show comparisons.
- A pictograph is mainly used to show categorical data. Pictographs are used to show frequency and compare items. However, the use of partial pictures can give misleading information.
  - Example:

The Types of Pets We Have



 = 1 student

- A circle graph is used for categorical and discrete numerical data. Circle graphs are used to show a relationship of the parts to a whole.
- All graphs must include a title, percent or number labels for data categories, and a key. A key is essential to explain how to read the graph. A title is essential to explain what the graph represents.
- A scale should be chosen that is appropriate for the data values being represented.
- Comparisons, predictions, and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions.
- The information displayed in different graphs may be examined to determine how data are or are not related, differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or "what could happen if" (inferences).

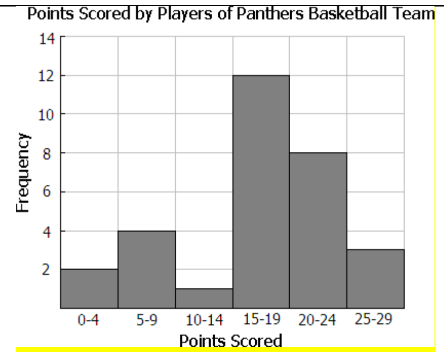
Histograms (SOL 7.9) (2 Days)

		# of	

Vocabulary	Big Ideas/VDOE Lesson Plans	Days	Essential Knowledge & Skills
histogram data circle graph line plot stem-and-leaf plot frequency title labels intervals inference comparison x-axis y-axis	**Big Ideas:** In many practical situations in a variety of areas such as business and science, observational data is gathered and graphed so that a unifying model can be determined to make predictions about unobserved input values. These predictions assist researchers in making important decisions about the situation of study. Therefore, it is also important that the reasonableness of predictions be addressed. **7.9a** • How can you construct a histogram? • When should you use a histogram to represent data? **7.9b** • How can a histogram help you infer or draw conclusions about a given set of data? **7.9c** • How can you interpret and compare data sets using data displays? **VDOE Lesson Plan**	**2**	• Collect, organize, and represent data in a histogram. • Make observations and inferences about data represented in a histogram. • Compare data represented in histograms with the same data represented in line plots, circle graphs, and stem-and-leaf plots.

Understanding the Standard

- A histogram is a graph that provides a visual interpretation of numerical data by indicating the number of data points that lie within a range of values, called a class or a bin. The frequency of the data that falls in each class or bin is depicted by the use of a bar. Every element of the data set is not preserved when representing data in a histogram.
- All graphs must include a title and labels that describe the data.
- Numerical data that can be characterized using consecutive intervals are best displayed in a histogram.
- Teachers should be reasonable about the selection of data values. Students should have experiences constructing histograms, but a focus should be placed on the analysis of histograms.
- A histogram is a form of bar graph in which the categories are consecutive and equal intervals. The length or height of each bar is determined by the number of data elements (frequency) falling into a particular interval.



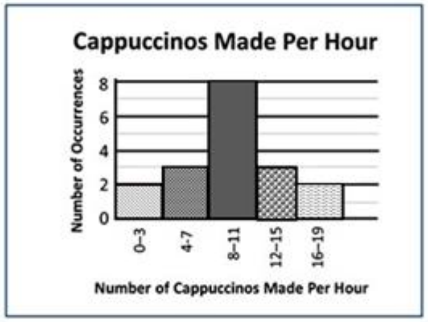
- A frequency distribution shows how often an item, a number, or range of numbers occurs. It can be used to construct a histogram.

Number of Cappuccinos Made per Hour at the Cafe

Number of Cups of Coffee	Tally	Frequency
0 - 3	||	2
4 – 7	|||	3
8 – 11	~~||||~~ |||	8
12 – 15	|||	3
16 – 19	||	2

To construct a histogram:

- Organize collected data into a table. Create one column for data range categories (bins), divided into equal intervals that will include all of your data (for example, 0-10, 11-20, 21-30), and another column for frequency.
  - Bins should be all the same size.
  - Bins should include all of the data.
  - Boundaries for bins should reflect the data values being represented.
  - Determine the number of bins based upon the data.
  - If possible, the number of bins created should be a factor the number of data values (e.g., a histogram representing 20 data values might have 4 or 5 bins).
- Create a graph. Mark the data range intervals on the *x*-axis (horizontal axis) with no space between the categories. Mark frequency on the *y*-axis (vertical axis), also in equal intervals.
- Plot the data. For each data range category (bin), draw a horizontal line at the appropriate frequency or marker. Then, create a vertical bar for that category reaching up to the marked frequency. Do this for each data range category (bin).



- Note: histograms may be drawn so that the bars are horizontal. To do this, interchange the *x*- and *y*-axis. Mark the data range intervals (bins) on the *y*-axis and the frequency on the *x*-axis. Draw the bars horizontally.

- Comparisons, predictions and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions. Data analysis helps describe data, recognize patterns or trends, and make predictions.
- There are two types of data: categorical and numerical. Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data. While students need to be aware of the differences, they do not have to know the terms for each type of data.
- Different types of graphs can be used to display categorical data. The way data is displayed is often dependent on what someone is trying to communicate.
- A line plot provides an ordered display of all values in a data set and shows the frequency of data on a number line. Line plots are used to show the spread of the data, to include clusters (groups of data points) and gaps (large spaces between data points), and quickly identify the range, mode, and any extreme data values.
- A circle graph is used for categorical and discrete numerical data. Circle graphs are used to show a relationship of the parts to a whole. Every element of the data set is not preserved when representing data in a circle graph.
- A stem and leaf plot is used for discrete numerical data and is used to show frequency of data distribution. A stem and leaf plot displays the entire data set and provides a picture of the distribution of data.
- Different situations or contexts warrant different types of graphs, and it helps to have a good knowledge of what graphs are available. Students can determine which graph makes the most sense to use based on the type of data provided and which graph can help them answer questions most easily.
- Comparing different types of representations (charts and graphs) provide students an opportunity to learn how different graphs can show different things about the same data. Following construction of graphs, students benefit from discussions around what information each graph provides.

The information displayed in different graphs may be examined to determine how data are or are not related, differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or "what could happen if" (inference).

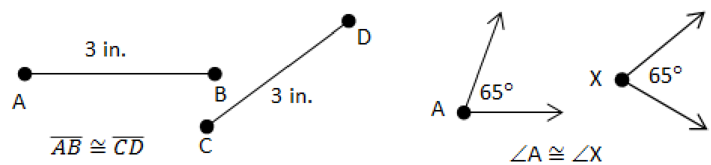
Congruency of Segments, Angles, and Polygons (SOL 6.9) (5 Days)

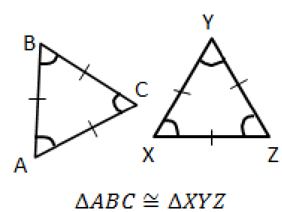
Vocabulary	Big Ideas/VDOE Lesson Plans	# of Days	Essential Knowledge & Skills

congruent symmetry line of symmetry segments side angles regular polygon non-congruent	**Big ideas:** • How does the line create 2 congruent parts when certain shapes are divided with a line of symmetry? • How do you determine if two figures are congruent? **VDOE Lesson Plans**	**5**	• Identify regular polygons. • Draw lines of symmetry to divide regular polygons into two congruent parts. • Determine the congruence of segments, angles, and polygons given their properties. • Determine whether polygons are congruent or non-congruent according to the measures of their sides and angles.

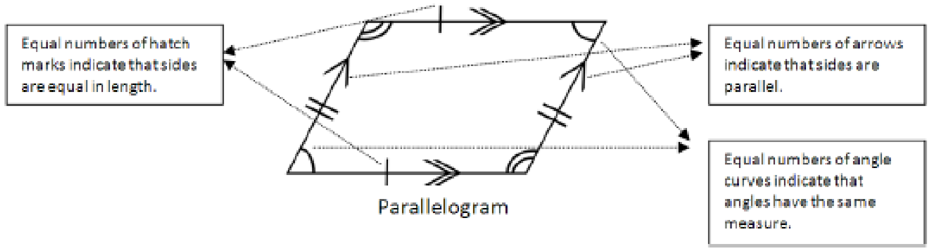
Understanding the Standard

- The symbol for congruency is $\cong$.
- Congruent figures have exactly the same size and the same shape. Line segments are congruent if they have the same length. Angles are congruent if they have the same measure. Congruent polygons have an equal number of sides, and all the corresponding sides and angles are congruent.
  - Examples:





- A polygon is a closed plane figure composed of at least three line segments that do not cross.
- A regular polygon has congruent sides and congruent interior angles.
- The number of lines of symmetry of a regular polygon is equal to the number of sides of the polygon.
- A line of symmetry divides a figure into two congruent parts, each of which is the mirror image of the other. Lines of symmetry are not limited to horizontal and vertical lines.
- Non-congruent figures may have the same shape but not the same size.
- Students should be familiar with geometric markings in figures to indicate congruence of sides and angles and to indicate parallel sides. An equal number of hatch (hash) marks indicate that those sides are equal in length. An equal number of arrows indicate that those sides are parallel. An equal number of angle curves indicate that those angles have the same measure. See the diagram below.



- The determination of the congruence or non-congruence of two figures can be accomplished by placing one figure on top of the other or by comparing the measurements of all corresponding sides and angles.
- Construction of congruent line segments, angles, and polygons helps students understand congruency.

Circles, Triangles, and Rectangles (SOL 6.7) (8 Days)

Vocabulary	Big Ideas/VDOE Lesson Plans	# of Days	Essential Knowledge & Skills
circle pi approximation circumference area radius diameter triangle base height rectangle length width perimeter	**Big ideas:** • Measurement concepts are used frequently in life. Objects have certain measurable attributes they can be quantified. Calculating circumference, perimeter and area is an application of these unit measurements. • Circles are present in objects that we may not realize. For example, the windshield wiper of a car rotates in a circular fashion, but does not necessarily rotate a full180°. Due to this, deciding which length wiper to replace on a car is an application of measurements of a circle. • Triangles and quadrilaterals can come in a variety of shapes and sizes, each of which has unique properties. These special properties are utilized when creating things such as floor plans, artwork, sculptures, logos, web design, etc. **6.7a** • What relationships exist among the radius, diameter, and circumference of a circle? **6.7b** • How does the circumference of a circle relate to the distance it rotates? (i.e. tire, ferris wheel, gears) • In what type of situations would you need to find the area of a circle? **6.7c** • How can patterns be used to determine formulas for area and perimeter? • How could you find the area of a square if you knew the perimeter? • How is the area formula for a triangle similar to the area	**8**	• Derive an approximation for pi (3.14 or $\frac{22}{7}$) by gathering data and comparing the circumference to the diameter of various circles, using concrete materials and computer models. • Solve problems, including practical problems, involving circumference and area of a circle when given the length of the diameter or radius. • Solve problems, including practical problems, involving area and perimeter of triangles and rectangles.

	formula of a rectangle? **VDOE Lesson Plans**		

Understanding the Standard

- The value of pi (π) is the ratio of the circumference of a circle to its diameter. Thus, the circumference of a circle is proportional to its diameter.
- The calculation of determining area and circumference may vary depending upon the approximation for pi. Common approximations for π include 3.14, $\frac{22}{7}$, or the pi (π) button on a calculator.
- Experiences in deriving the formulas for area, perimeter, and volume using manipulatives such as tiles, one-inch cubes, graph paper, geoboards, or tracing paper, promote an understanding of the formulas and their use.
- Perimeter is the path or distance around any plane figure. The perimeter of a circle is called the circumference.
- The circumference of a circle is about three times the measure of its diameter.
- The circumference of a circle is computed using $C = \pi d$ or $C = 2\pi r$, where d is the diameter and r is the radius of the circle.
- The area of a closed curve is the number of nonoverlapping square units required to fill the region enclosed by the curve.
- The area of a circle is computed using the formula $A = \pi r^2$, where r is the radius of the circle.
- The perimeter of a square whose side measures s can be determined by multiplying 4 by s ($P = 4s$), and its area can be determined by squaring the length of one side ($A = s^2$).
- The perimeter of a rectangle can be determined by computing the sum of twice the length and twice the width ($P = 2l + 2w$, or $P = 2(l + w)$), and its area can be determined by computing the product of the length and the width ($A = lw$).
- The perimeter of a triangle can be determined by computing the sum of the side lengths ($P = a + b + c$), and its area can be determined by computing $\frac{1}{2}$ the product of base and the height ($A = \frac{1}{2}bh$).

Quarter 3: (43 Instructional Days)

4th Quarter

Central Tendency (SOL 6.11) (6 Days)

Vocabulary	Big Ideas/VDOE Lesson Plans	# of Days	Essential Knowledge & Skills
data set mean fair share balance point line plot measure of center median mode	**Big ideas:** Measures of center are used to describe sets of data. Different measures of center can be used to interpret sets of data and make sense of how data is distributed. Batting averages, median home prices, and frequency tables all use measures of center to describe data and can be used to predict outcomes. **6.11a** • How is the mean represented graphically as a balance point? **6.11b** • In which situations would the mean, median, or mode be the best representation of the data?	**6**	• Represent the mean of a set of data graphically as the balance point represented in a line plot. • Determine the effect on measure of center when a single value of a data set is added, removed, or changed.

	• What effect does adding or removing one piece of data have on the mean? • What effect does adding or removing one piece of data have on the mode? • What effect does adding or removing one piece of data have on the median? **VDOE Lesson Plans**		

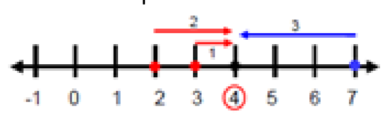
Understanding the Standard

- Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data.
- Measures of center are types of averages for a data set. They represent numbers that describe a data set. Mean, median, and mode are measures of center that are useful for describing the average for different situations.

  Mean may be appropriate for sets of data where there are no values much higher or lower than those in the rest of the data set.
  Median is a good choice when data sets have a couple of values much higher or lower than most of the others.
  Mode is a good descriptor to use when the set of data has some identical values, when data is non-numeric (categorical) or when data reflects the most popular item.
- Mean can be defined as the point on a number line where the data distribution is balanced. This requires that the sum of the distances from the mean of all the points above the mean is equal to the sum of the distances from the mean of all the data points below the mean. This is the concept of mean as the balance point.
  - Example: Given the data set:

  2, 3, 4, 7

  The mean value of 4 can be represented on a number line as the balance point:



- The mean can also be found by calculating the numerical average of the data set.
- In 5th grade mathematics, mean is defined as fair share.
- Defining mean as the balance point is a prerequisite for understanding standard deviation, which is addressed in high school level mathematics.
- The median is the middle value of a data set in ranked order. If there are an odd number of pieces of data, the median is the middle value in ranked order. If there is an even number of pieces of data, the median is the numerical average of the two middle values.
- The mode is the piece of data that occurs most frequently. If no value occurs more often than any other, there is no mode. If there is more than one value that occurs most often, all these most-frequently-occurring values are modes. When there are exactly two modes, the data set is bimodal.

SOL Review (25 Days)

Proportional Relationships (SOL 7.3) (5 Days)

Vocabulary	Big Ideas/VDOE Lesson Plans	# of Days	Essential Knowledge & Skills

proportional relationships ratio table US Customary metric benchmark tips tax discounts equivalent ratio product of extremes product of means percent unit rate mass weight	**Big ideas:** Proportional reasoning involves thinking about relationships and making comparisons of quantities or values. People use proportional reasoning to calculate best buys, taxes and investments, to work with drawings and maps, to measure or exchange money, to adjust recipes, or to create various concentrations of mixtures and solutions. • How do we know if the relationship between any two numbers is proportional? • Is there more than one way to solve a proportion? Explain. • In what type of scenarios do we use proportions to solve problems? • How do you mentally calculate 10% of any number? • How can 10% be used as a benchmark to calculate other percentages? • What strategies do you use to solve problems with tax, tip, and discount? • How do you use proportions to convert between the U.S. Customary and the metric systems? • How do you use proportions to find scale factor and create a scale drawing? **VDOE Lesson Plans**	**5**	• Given a proportional relationship between two quantities, create and use a ratio table to determine missing values. • Write and solve a proportion that represents a proportional relationship between two quantities to find a missing value. • Apply proportional reasoning to convert units of measurement within and between the U.S. Customary System and the metric system when given the conversion factor. • Apply proportional reasoning to solve practical problems, including scale drawings. Scale factors shall have denominators no greater than 12 and decimals no less than tenths. • Using 10% as a benchmark, compute 5%, 10%, 15%, or 20% of a given whole number. • Using 10% as a benchmark, compute 5%, 10%, 15%, or 20% in a practical situation such as tips, tax, and discounts. • Solve problems involving tips, tax, and discounts. Limit problems to only one percent computation per problem.

Understanding the Standard

- A proportion is a statement of equality between two ratios. A proportion can be written as $\frac{a}{b} = \frac{c}{d}$, a:b = c:d, or a is to b as c is to d.
- Equivalent ratios arise by multiplying each value in a ratio by the same constant value. For example, the ratio of 3:2 would be equivalent to the ratio 6:4 because each of the values in 3:2 can be multiplied by 2 to get 6:4.
- A ratio table is a table of values representing a proportional relationship that includes pairs of values that represent equivalent rates or ratios.
- A proportion can be solved by determining the product of the means and the product of the extremes. For example, in the proportion a:b = c:d, a and d are the extremes and b and c are the means. If values are substituted for a, b, c, and d such as 5:12 = 10:24, then the product of extremes (5 · 24) is equal to the product of the means (12 · 10).
- In a proportional relationship, two quantities increase multiplicatively. One quantity is a constant multiple of the other.
- A proportion is an equation which states that two ratios are equal. When solving a proportion, the ratios may first be written as fractions.
  - Example: A recipe for oatmeal cookies calls for 2 cups of flour for every 3 cups of oatmeal. How much flour is needed for a larger batch of cookies that uses 9 cups of oatmeal? To solve this problem, the ratio of flour to oatmeal could be written as a fraction in the proportion used to determine the amount of flour needed when 9 cups of oatmeal is used. To use a proportion to solve for the unknown cups of flour needed, solve the proportion: $\frac{2}{3} = \frac{x}{9}$.
    To use a table of equivalent ratios to find the unknown amount, create the table:

flour (cups)	2	4	?
oatmeal (cups)	3	6	9

To complete the table, we must create an equivalent ratio to 2:3; just as 4:6 is equivalent to 2:3, then 6 cups of flour to 9 cups of oatmeal would create an equivalent ratio.

- A proportion can be solved by determining equivalent ratios.

- A rate is a ratio that compares two quantities measured in different units. A unit rate is a rate with a denominator of 1. Examples of rates include miles/hour and revolutions/minute.
- Proportions are used in everyday contexts, such as speed, recipe conversions, scale drawings, map reading, reducing and enlarging, comparison shopping, tips, tax, and discounts, and monetary conversions.
- A multistep problem is a problem that requires two or more steps to solve.
- Proportions can be used to convert length, weight (mass), and volume (capacity) within and between measurement systems. For example, if 1 inch is about 2.54 cm, how many inches are in 16 cm?

$$\frac{1\ inch}{2.54\ cm} = \frac{x\ inch}{16\ cm}$$

$$2.54x = 1 \cdot 16$$

$$2.54x = 16$$

$$x = \frac{16}{2.54}$$

$x = 6.299$ or about 6.3 inches

- Examples of conversions may include, but are not limited to:
  - Length: between feet and miles; miles and kilometers
  - Weight: between ounces and pounds; pounds and kilograms
  - Volume: between cups and fluid ounces; gallons and liters
- Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes depending on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?").
- When converting measurement units in practical situations, the precision of the conversion factor used will be based on the accuracy required within the context of the problem. For example, when converting from miles to kilometers, we may use a conversion factor of 1 mile ≈ 1.6 km or 1 mile ≈ 1.609 km, depending upon the accuracy needed.
- Estimation may be used prior to calculating conversions to evaluate the reasonableness of a solution.
- A percent is a ratio in which the denominator is 100.

- Proportions can be used to represent percent problems as follows:

$$\frac{percent}{100} = \frac{part}{whole}$$

Similar Figures (SOL 7.5) (4 Days)

Vocabulary	Big Ideas/VDOE Lesson Plans	# of Days	Essential Knowledge & Skills
similar corresponding proportional scale factor angles sides ratio congruent	**Big ideas:** Similar figures are widespread in our everyday lives. For example, using shadows of objects at a measureable height and properties of similar triangles, we can find the height of a tree, or any other extremely tall object. If the properties of similar figures are not applied in the zoom lens of a camera, pictures we take every day would be distorted. • What is the relationship between corresponding angles in two similar figures? • What is the relationship between corresponding sides in two similar figures? • What are the properties of similar figures? • How do you use proportions to find side lengths of similar figures? • What is a similarity statement? • How do you use proportions to show the relationships between corresponding sides of two similar figures? • How can you determine a missing angle measure when given two angle measures within similar figures? • How do you use symbols to represent congruent angles? **VDOE Lesson Plan**	**4**	• Identify corresponding sides and corresponding congruent angles of similar quadrilaterals and triangles. • Given two similar quadrilaterals or triangles, write similarity statements using symbols. • Write proportions to express the relationships between the lengths of corresponding sides of similar quadrilaterals and triangles. • Solve a proportion to determine a missing side length of similar quadrilaterals or triangles. • Given angle measures in a quadrilateral or triangle, determine unknown angle measures in a similar quadrilateral or triangle.

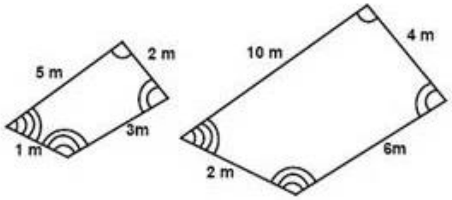
Understanding the Standard

- Similar polygons have corresponding sides that are proportional and corresponding interior angles that are congruent.
- Similarity has practical applications in a variety of areas, including art, architecture, and the sciences.
- Similarity does not depend on the position or orientation of the figures.
- Congruent polygons have the same size and shape. Corresponding angles and sides are congruent.
- Congruent polygons are similar polygons for which the ratio of the corresponding sides is 1:1. However, similar polygons are not necessarily congruent.
- The symbol ~ is used to represent similarity. For example, $\Delta ABC \sim \Delta DEF$.
- The symbol ≅ is used to represent congruence. For example, $\angle A \cong \angle B$
- Similarity statements can be used to determine corresponding parts of similar figures such as:
  Given: $\Delta ABC \sim \Delta DEF$

$\angle A$ corresponds to $\angle D$

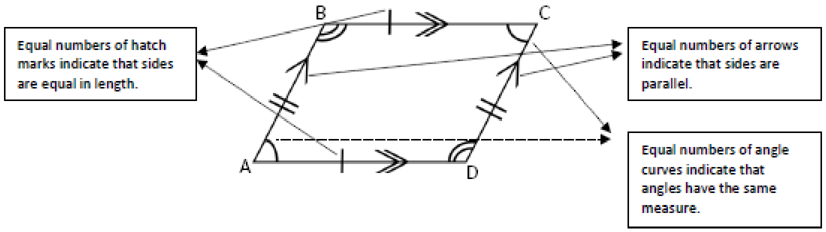
$\overline{AB}$ corresponds to $\overline{DE}$

- A proportion representing corresponding sides of similar figures can be written as $\frac{a}{b} = \frac{c}{d}$.
  - Example: Given two similar quadrilaterals with corresponding angles labeled, write a proportion involving corresponding sides.



$\frac{5}{10} = \frac{2}{4}$ or $\frac{5}{10} = \frac{3}{6}$ or $\frac{1}{2} = \frac{2}{4}$ are some of the ways to express the proportional relationships that exist.

- The traditional notation for marking congruent angles is to use a curve on each angle. Denote which angles are congruent with the same number of curved lines. For example, if $\angle A$ is congruent to $\angle C$, then both angles will be marked with the same number of curved lines.



- Congruent sides are denoted with the same number of hatch (or hash) marks on each congruent side. For example, a side on a polygon with 2 hatch marks is congruent to the side with 2 hatch marks on a congruent polygon or within the same polygon.